THE VERIFICATION OF THE MATHEMATICAL MODEL AND THE OPTIMIZATION OF THE MUNICIPAL SOLID WASTE COMPOSTING PROCESS[*]

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The aims of this work were to develop a mathematical model for composting municipal solid waste (MSW), to verify the proposed model with independent experimental data and to determine the optimum values for the initial moisture content and airflow. Separated MSW (with the addition of poultry manure, mature compost and sawdust) was used as materials for laboratory simulation. The experiment was conducted in three laboratory reactors (35 L) with forced aeration. Experimental data from one reactor were used to estimate five kinetic parameters in the proposed model and the verification of the model was performed using experimental data from the two reactors. Kinetics is based on the decomposition rate of organic matter. The estimation of kinetic parameters was done using data for four measured dynamic variables (organic matter conversion, O_2 and CO_2 concentration, and substrate temperature). Mathematical model was implemented in MATLAB. The proposed kinetic model is: $k_T = 0.0152 \cdot \left[1.0176 - 2.8001^{(-0.002 \cdot (T-20))} \right]$ with the reaction order 1.6. Comparisons of experimental and simulation results showed good agreement, especially in the mesophilic-thermophilic phase of the process, considering a high heterogeneous system. Optimum values for the initial moisture content in the material (45%) and airflow (0.50 $m^3 h^{-1} kg^{-1}_{OT}$) were estimated using numerical simulations for different initial conditions.

Key words: verification, mathematical model, MSW composting, optimization

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INTRODUCTION

Composting process is a very complex process, during which the organic matter breaks down in a series of biochemical reactions, with mature compost as the final product, with different composition and quality, which depends on the substrate and the parameters that are essential for the process. The composting process is very important in the recycling of important nutrients that are lost with waste in landfills which have multiple adverse effects on the environment. In order to develop a process that would lead to a more efficient conversion of organic matter and the reduction of the negative impact of waste on the environment, mathematical modeling provides opportunities for the simulation and optimization of the process, which significantly reduces efforts in a design of reactor and non-reactor systems for the composting process. Most of the existing models use parameter values from available literature, or few dynamic variables for the estimation of kinetic parameters, and the verification of the model was carried out either only with one or two experimentally measured dynamic state variables, or not carried out at all. Only few researchers performed the optimization of MSW composting process. The aims of this research are to develop a mathematical model for composting of MSW based on the existing mathematical models and the experimental data obtained in laboratory conditions, to perform numerical simulation of the composting process and finally, to determine the optimum values of the initial moisture content and airflow.

EXPERIMENTAL PART

Description and limitations of the model

The model is based on the basic principles of kinetics, stoichiometry, mass balance and heat balance in a three-phase system (solid-liquid-gas). The substrate (organic matter, inorganic matter and water) is degraded through biochemical reactions in the presence of O_2 to form CO_2, NH_3 and H_2O vapor. With respect to the substrate as a reactant, a reactor can be modeled as a batch, while the considered present gas reactor model can be approximated by a model of a continuous stirred tank reactor in an unsteady state. In developing the model, the following assumptions are taken: the volume of the gas phase, pressure in the reactor and the air flow are constant; the gas phase is saturated with water vapor; liquid-solid and gas phases have equal temperatures, substrate is a homogenous mixture with the same composition in every point and elemental composition of organic matter in the substrate at the beginning of the process, are known. Proposed kinetics is n-th order, where the reaction rate is equal to the rate of organic matter degradation. A specific rate constant is a function of temperature, pH, O_2, moisture content and free air space.

As a basis for describing the effect of temperature on the reaction rate constant, the expression developed by reference [1] was used:

$$k_T = a \cdot (1 - e^{-c \cdot (T - T_0)}) \tag{1}$$

where : a,c-constant, T-substrate temperature (° C), T_0-reference temperature, 20°C.

In this model, the proposed a modified form of the expression (1) is:

$$k_T = a \cdot (b - c^{d \cdot (T-20)})$$ (2)

where: a, b, c, d-constants to be determined, the with reaction order n. More details can be found in the reference [2]. Equations for the calculation of mass three dissolved gases (O_2, CO_2, NH_3) in water of the material for composting, the amount of gas in the gas phase (O_2, CO_2, NH_3, N_2 and H_2O), the temperature of the gas and solid-liquid phase are presented in references [2] and [3]. Mathematical model consists of 12 nonlinear differential equations with corresponding linear and nonlinear algebraic equations.

LABORATORY SIMULATION

Materials
Separated MSW (food waste, fruits and vegetables, paper and cardboard, garden waste) was used in the experiment with the addition of poultry manure, mature compost and sawdust. Basic physical-chemical characteristics of separated MSW, poultry manure, compost and sawdust are shown in Table 1.

Table 1. Basic physico-chemical characteristics separated MSW, poultry manure, mature compost and sawdust, before mixing (three measurements, mean value)

Material	Total solids (% w.b.)	Moisture (% w.b.)	Organic matter (% d.b.)	Ash (% d.b.)	pH	Electrical conductivity (dS m^{-1})	C/N
MSW	40.17	59.83	91.69	8.31	4.98	1.19	50.94
Poultry manure	28.97	71.03	78.89	21.11	8.31	3.77	43.83
Sawdust	89.97	10.03	99.9	0.1	5.31	0.24	55.50
Compost	67.49	32.51	40.31	59.69	6.92	0.35	22.39

w.b. – wet basis, d.b. – dry basis

Design of experiment and applied methods
The study was carried out in three reactors with a term of 22 days and the results of this the experiment were used to determine the kinetic parameters in the proposed mathematical model. For all three reactors, air flows were calculated by mass of organic matter in accordance with published recommendations ([3], [4]). Mass compounds that reactors were filled with were: 7.90 kg of the mixture in the first reactor (60% MSW, 20% poultry manure, and per 10% sawdust and compost), 10.80 kg of the mixture in the second reactor (50% MSW, 33.33% poultry manure, and per 8.33% sawdust and compost), 12.65 kg of the mixture in the third reactor (42.86% MSW, 42.86% poultry manure, and per 7.14% sawdust and compost). During the entire process, the substrate temperature was continuously measured (every 15 minutes) in all three reactors, as well as the ambient temperature. Concentrations of O_2 and CO_2 (in vol% sb), moisture and total solids [5], organic matter and ash [5], pH and

electrical conductivity [5] were measured daily. Table 2 gives basic physico-chemical properties for the mixture of MSW, poultry manure, sawdust and compost in all three reactors.

Table 2 Characterization of a mixture of MSW, poultry manure, sawdust and mature compost in all reactors (three measurements, mean value)

Reactor	Moisture (% w.b.)	Total solids (% w.b.)	Organic matter (% d.b.)	Ash (% d.b.)	pH	Electrical conductivity (dS m^{-1})	C/N
1	45.56	54.44	87.53	12.47	5.17	1.50	40.19
2	58.17	41.83	85.61	14.39	6.27	2.17	36.59
3	60.38	39.62	83.89	16.11	6.37	2.62	33.77

w.b. – wet basis, d.b. – dry basis

More details about the experiment can be found in the reference[2]. Laboratory reactors were specially designed, and they were made of stainless steel (diameter 0.55, height 0.36 m). The reactors were isolated with polyethylene layer, the thickness of 10 mm and equipped with mixers. Figure 1 shows the reactor system that was used to perform the experiment.
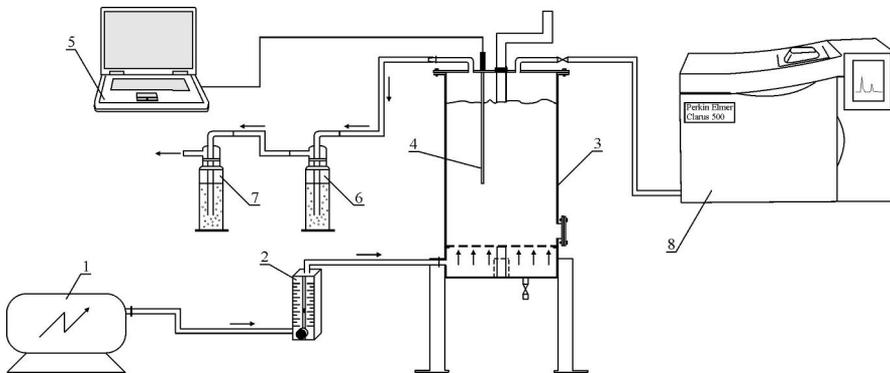

Figure 1 A schematic diagram of the reactor system for aerobic composting. (1) air compressor, (2) air flow meter, (3) reactor, (4) thermocouple, (5) portable computer, (6) bottle with NaOH solution, (7) bottle with H_3BO_3 solution, (8) gas chromatograph.

Numerical simulation

A mathematical model was implemented in the numerical software package MATLAB[6] and as a numerical method for solving systems of differential equations the ODE23s solver was used. Marquardt method [7] was used for determination of the kinetic parameters in the model. The model was fitted with data for the substrate temperature, the concentrations of O_2 and CO_2, and the organic matter mass. The data used for numerical simulations can be found in the reference [2].

RESULTS AND DISCUSSION

Parameter estimation was based on the experimental data (organic matter conversion, CO_2 and O_2 concentrations and substrate temperature) from the third reactor (R3), which are shown in Figures 2, 3, 4 and 5.

Based on these experimental data, the kinetic parameters were calculated: the reaction order is 1.6, and kinetic model is: $k_T = 0.0152 \cdot \left[1.0176 - 2.8001^{(-0.002 \cdot (T-20))} \right]$.
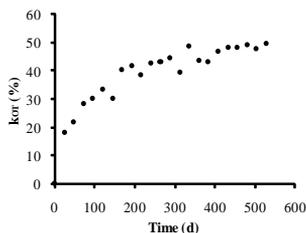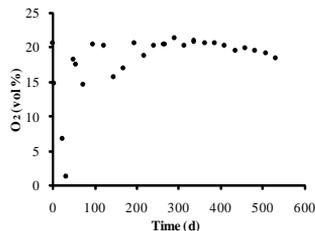

Figure 2. Experimental data for the organic matter conversion

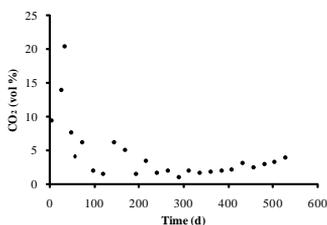Figure 3. Experimental data for O_2 concentration

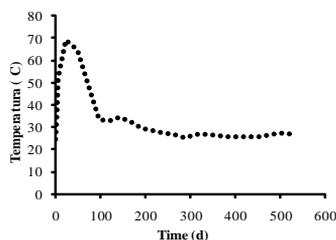Figure 4. Experimental data for CO_2 concentration

Figure 5. Experimental data for the substrate temperature

The organic matter conversion was the greatest during the first eight days, after which the percentage of degradation slowly decreased, which indicated that sugars, fats and proteins were degraded and therefore, cellulose, hemicellulose and lignin remained in the substrate. Better degradation of the organic matter in the second reactor (Fig.7) can be explained by the fact that the content of a mature compost (inoculum) and poultry manure was higher compared to the first reactor. In the first reactor (Fig.6) the degradation was a little slower so the lag phase lasted for about one day. Similar observations correspond to the literature data ([8], [9]). Deviations between experimental and simulated data can be explained by the fact that it is a very heterogeneous system [10]. The comparison of the model with experimental data for the conversion of organic matter from the first (R1) and second reactor (R2), are shown in Figures 6 and 7.
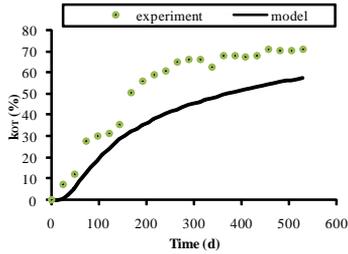
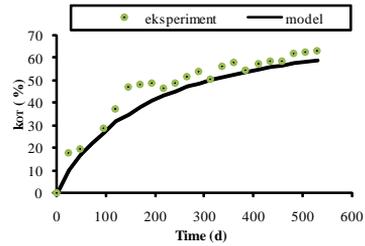Figure 6. Model verification for
the organic matter conversion (R1)

Figure 7. Model verification for
the organic matter conversion (R2)

One type of microorganisms breaks down a heterogeneous substrate, such that a single type of the substrate, was major simplification used in the model. The results of the model comparison with experimental data for O_2 concentration in R1 and R2 are shown in Figures 8 and 9. The concentrations of O_2 for the model and experiment results showed good agreement during the entire duration of the process in both reactors. Major discrepancy was observed in R2, after 9-th day, which is probably a result of decomposition, various kinds of components which significantly slowed the process down because a mesophilic-thermophilic phase was already completed, and the consumption oxygen was much lower.
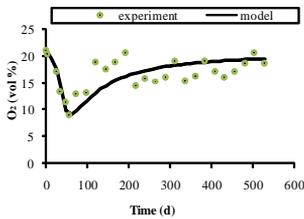

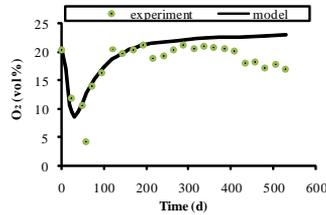Figure 8. Model verification for
O_2 concentration (R1)

Figure 9. Model Verification for
O_2 concentration (R2)

Also, during the organic matter decomposition process, porosity of the materials reduced, O_2 was not retained in the material, so that O_2 concentration increased in the outlet gas from the reactor[11]. As O_2 concentration was measured at the exit that increased deviations between the experimental and model results. Researchers had similar conclusions [1].

The results of the model agreement with experimental data for CO_2 concentration in R1 i R2 are shown in Figures 10 and 11. As the concentration of O_2, the largest deviation in results for CO_2 concentration in both reactors was observed immediately after reaching a thermophilic peak.
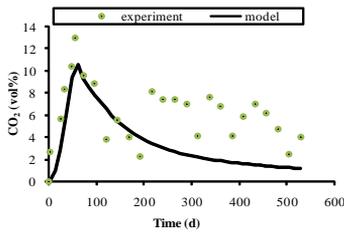
Figure 10. Model verification
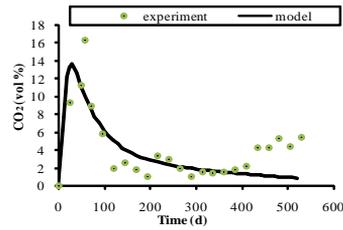for CO_2 concentration (R1)

Figure 11. Model verification
for CO_2 concentration (R2)

Such deviations could be explained by mixing the material, because it helps make reallocation of the substrate and water and improves the aeration of the mass, whereby there was the increased degradation of the substrate. Similar observations can be found in the references [12] and [13]. The results of the model agreement with experimental data for the substrate temperature from R1 and R2 are shown in Figures 12 and 13. he comparison of experimental and simulation results for the substrate temperature showed good agreement during the mesophilic and thermophilic phase of the process. After reaching a thermophilic peak, composting mass was cooled, so the model predicted a slower cooling in comparison to experimental data, which can be explained by the fact that the model did not take into account all biochemical reactions that otherwise occur in the actual process. In the experiment, there was a decrease of the moisture content in the material, which reduced a thermal capacity of materials, leading to a more cooled mass. A maximum temperature difference was observed in the cooling phase.
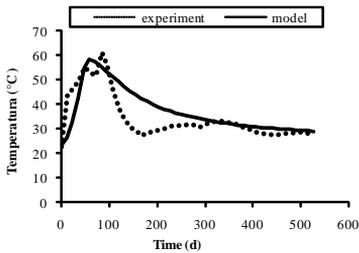

Figure 12. Model verification for
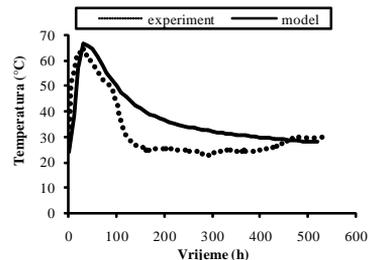the substrate temperature (R1)

Figure 13. Model verification for
the substrate temperature (R2)

Similar deviations were observed in previous studies [1] and [14]. Some differences could be explained by variations in the ambient temperature (day and night). From Figures 12 and 13 it can be seen that R2 peak temperature difference was only 1.7°C, while in R1, that value was 2.6°C. Similar values were found by [15]. Although it is considered that temperatures above 60°C lead to the inactivation of microorganisms, it can be said that this limit can be shifted depending on the composition of the substrate and aeration, which promotes the removal of the excess heat from the mass. Numerical simulations provide an excellent opportunity to determine the effect of the basic

process factors such as the initial moisture content and airflow. Series of simulations led to optimal values of these factors. Table 3 shows the values for maximum conversion of organic matter and maximum substrate temperature for different values of the initial moisture content.

Table 3. The influence of the initial moisture content at maximum conversion
of organic matter and maximum substrate temperature

Initial moisture content in substrate (%)	Max. conversion of organic matter (%)	Max. temperature of substrate (°C)
40	69.86	77.56
45	70.06	78.05
50	69.33	77.98
60	61.91	69.56

The calculated optimal value of the initial moisture content was 45%, which is consistent with the results obtained by [1] (shown in Table 3). A very small initial value of the moisture content could lead to dehydration early in the process, which would stop the process, giving physically stable but microbiologically unstable compost [16]. Table 4 shows maximum values of the organic matter conversion and substrate temperature for different values of airflow. Numerical simulations showed that the optimum value of airflow was also experimental value, 0.50 $m^3h^{-1}kg^{-1}_{OT}$, because with this value the airflow highest organic matter conversion and substrate temperature were achieved. The calculated value of the flow rate approximates published literature values of other authors for different composting materials ([4], [17], [3]).

Table 4. The effect of the air flow for maximum conversion of
the organic matter and maximum substrate temperature

Airflow ($m^3 h^{-1} kg_{OT}^{-1}$)	Max. conversion of organic matter (%)	Max. temperature of substrate (°C)
0.42	61.06	69.49
0.50	61.91	69.56
0.58	59.14	69.53
0.75	61.87	65.65

CONCLUSIONS

The experiment was carried out with composting of separated MSW with the addition of poultry manure, mature compost and sawdust, for a period of 22 days in three specially designed laboratory reactors under controlled conditions. Based on the data obtained by the experiment (R3), five kinetic parameters in the proposed mathematical

model were obtained. The verification of the proposed model was performed with independent data obtained in the remaining two reactors (organic matter conversion, O_2 and CO_2 concentrations and substrate temperature). Although it is a very heterogeneous system, the comparison of the experimental and simulation showed very good agreement during the mesophilic-thermophilic phase of the process, while the cooling phase observed some deviations, mainly due to degradation of cellulose, hemicellulose, lignin, etc., which has shown that the developed model can simulate the process of aerobic composting of MSW well, with the addition of poultry manure, mature compost and sawdust. Considering two objective functions (organic matter conversion, and substrate temperature) the ptimal value of the initial moisture content (45%) and airflow (0.50 m^3 h^{-1} kg^{-1}_{OT}) were calculated. These values are consistent with the published data. In future, the model should include more kinetic parameters for a better description of actual processes, as well as different types of microorganisms that break down different types of substrates or the model should be divided into two parts: a fast degradation model for describing degradation components such as sugars, proteins and fats and a slow degradation model for describing degradation components such as cellulose, hemicellulose and lignin.

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IZVOD

VERIFIKACIJA MATEMATIČKOG MODELA I OPTIMIZACIJA PROCESA KOMPOSTIRANJA KOMUNALNOG KRUTOG OTPADA

(Original scientific paper)

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Ciljevi rada su bili: da se razvije novi matematički model za proces kompostiranja komunalnog krutog otpada, da se verificira predloženi model sa nezavisnim eksperimentalnim podacima i da se odrede optimalne vrijednosti za početni sadržaj vlage i protok zraka. Za eksperiment je korišten separirani komunalni kruti otpad uz dodatak peradarskog gnoja, zrelog komposta i piljevine. Eksperiment je izveden u tri laboratorijska reaktora (35 l) sa prisilnom aeracijom. Eksperimentalni podaci iz jednog reaktora su iskorišteni za procjenu pet kinetičkih parametara u predloženom modelu, a verifikacija modela je izvršena na osnovu eksperimentalnih podataka iz preostala dva reaktora. Kinetika procesa je zasnovana na brzini razgradnje organskih tvari. Procjena kinetičkih parametara je izvršena na osnovu četiri mjerene dinamičke varijable (konverzija organskih tvari, koncentracija O_2 i CO_2, temperatura supstrata). Matematički model je implementiran u programskom paketu MATLAB. Predložen je kinetički model: $k_T = 0.0152 \cdot \left[1.0176 - 2.8001^{(-0.002 \cdot (T-20))} \right]$, sa redom reakcije 1.6 Poređenja eksperimentalnih i simulacijskih rezultata pokazala su dobro slaganje, posebno u dijelu mezofilno-termofilne faze procesa iako se radi o jako heterogenom sistemu. Optimalne vrijednosti za početni sadržaj vlage u materijalu (45 %) i protok zraka (0.50 $m^3h^{-1}kg^{-1}_{OT}$) određene su korištenjem numeričkih simulacija za različite početne uslove.

Ključne riječi: verifikacija, matematički model, kompostiranje komunalnog krutog otpada, optimizacija